Control Systems

(Prof. Casella)

Written Exam – July 2^{nd}, 2015

Surname:...

Name: ....

Reg. Number:...............................

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Notices:

- This booklet is comprised of 7 sheets – Check that it is complete and fill in the cover.
- Write your answers in the blank spaces with short arguments, including only the main steps in the derivation of the results.
- You are not allowed to leave the classroom unless you hand in the exam paper or withdraw from the exam.
- You are not allowed to consult books or lecture notes of any kind.
- Please hand in only this booklet at the end of the exam – no loose sheets.
- The clarity and order of your answers will influence how your exam is graded.

Question 1

With reference to feed-back control systems, explain why the input-output dynamic behaviour of the plant to be controlled is crucial for the system performance. Also explain why, on the other hand, a very precise knowledge of that dynamic behaviour is not required, if the controller is designed properly.

Question 2

Briefly explain why the stability of a linear system with transfer function $G(s)$ is completely determined by its poles. Then, state precisely under which conditions such system will be asymptotically stable, simply stable, or unstable.

Question 3

Consider a one-dimensional spring-mass mechanical system, in which a body of mass M is connected to the ground by a spring and by a damper, and subject to an external force F. Contrary to a standard spring-mass harmonic oscillator, the spring has a nonlinear force-displacement characteristic, where the force grows more than proportionally to the spring deformation.

The equations describing the system are the following, where x is the body displacement, v is the body velocity, F_s is the spring force, F_d is the damper force, K and x_0 are the spring parameters, and h is the damper friction coefficient.

$$M\ddot{x} = F_s + F_d + F$$

$$F_s = -\frac{K}{x_0^2} x\left(x^2 + x_0^2\right)$$

$$F_d = -hv$$

3.1 Write down the state and output equations in standard state-space form, considering F as input and x, v as outputs

3.2 Compute the equilibrium conditions for the system

3.3 Write down the system's linearized equations around a generic equilibrium

3.4 Assuming the equilibrium value of F is zero, compute the transfer functions of the system between the deviations of the input ΔF and the deviations of the outputs Δx and Δv.

3.5 Determine under which conditions the system shows an oscillatory behaviour in response to a unit step change of the input ΔF, then plot the qualitative diagrams of the corresponding $\Delta x(t)$ and $\Delta \mathrm{v}(t)$.

3.6 Assume now that the equilibrium value of F is no longer zero, but rather that the spring is pre-loaded by a suitable force value such that the corresponding equilibrium displacement is x_0. Explain how the plots determined at the previous point change, all other parameters being unchanged.

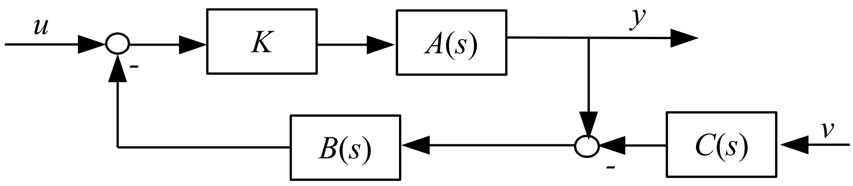
Question 4

Considering the following block diagram

$$A(s)=\frac{10}{1+s}$$

$$B(s)=\frac{1}{1+s}$$

$$C(s)=\frac{5}{1+s}$$



4.1 Compute the transfer functions between the inputs *u* and *v* and the output *y*.

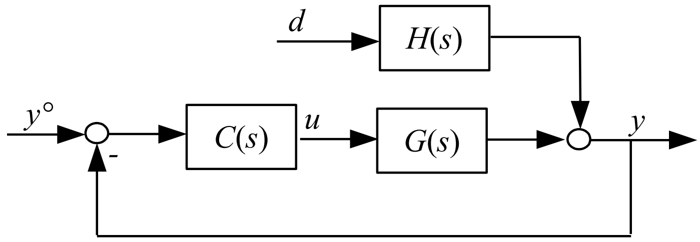
4.2 Determine for which values of the parameter *K* the system shows exponentially diverging oscillations in response to step changes of the inputs.

Question 5

Consider the following control system, where the unit of time constants is the second:

$$G(s)=\frac{500}{s(1+100s)(1+30s)(1+10s)(1+5s)}$$

$$H(s)=\frac{10}{s(1+100s)}$$



5.1 Design a PI or PID controller with a bandwidth of 0.003 rad/s and at least 60° phase margin.

5.2 Design a PID controller with the highest bandwidth you can get for a phase margin of 45°, assuming $N = 10$ for the derivative action.

5.3 Plot the qualitative diagrams of the response of the controlled output y to a step change in the disturbance d for the two controllers designed at points 5.1 and 5.2.

5.4 Determine the asymptotic amplitude of the oscillations of the manipulated variable u corresponding to a disturbance $d = \sin(2t)$ for the two controllers designed at points 5.1 and 5.2.