Control Systems

(Prof. Casella)

Written Exam – July 22^{nd}, 2015

Surname:..

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Reg. Number:...............................

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Notices:

- This booklet is comprised of 7 sheets – Check that it is complete and fill in the cover.
- Write your answers in the blank spaces with short arguments, including only the main steps in the derivation of the results.
- You are not allowed to leave the classroom unless you hand in the exam paper or withdraw from the exam.
- You are not allowed to consult books or lecture notes of any kind.
- Please hand in only this booklet at the end of the exam – no loose sheets.
- The clarity and order of your answers will influence how your exam is graded.

Question 1

Define the frequency response of a linear, time-invariant system. Then, state how it can be used to determine the system response to a sinusoidal input in the time domain.

Question 2

Illustrate the phenomenon of integral wind-up and how it can be avoided.

Question 3

Consider a thermo-hydraulic system in which a volume V of well-mixed fluid exchanges heat with a wall that is kept at a constant temperature T_w by some external means. A mass flow rate w_i of fluid at temperature T_i enters the volume, and a mass flow rate of fluid w_o leaves it. The heat transfer coefficient between the fluid and the wall is proportional to $w_o^{0.8}$. It is assumed for simplicity that the fluid has constant density and that $dh = de = c\,dT$.

The equations describing the system are the following, where M is the constant fluid mass, T the fluid temperature, c the constant fluid specific heat capacity, Q the heat flow to the fluid, and w_{nom} the nominal value of the flow rate:

$$Mc\frac{dT}{dt} = w_i c T_i - w_o c T + Q$$

$$w_i = w_o$$

$$Q = UA(T_w - T)$$

$$UA = UA_{nom}\left(\frac{w_o}{w_{nom}}\right)^{0.8}$$

3.1 Write down the state and output equations in standard state-space form, considering w_i and T_i as inputs, T and Q as outputs

3.2 Compute the equilibrium conditions for the system, assuming $w_o = w_{nom}$

3.3 Write down the system's linearized equations around the previously found equilibrium

3.4 Compute the transfer functions of the system between the deviations of the input ΔT_i and the deviations of the outputs ΔT and ΔQ and write them down in gain / time constants form.

3.5 Plot the qualitative diagrams of the unit step response of the transfer functions computed at point 3.4

3.6 Assume $\Delta T_i = sin(\omega t)$. Determine for which values of ω the corresponding oscillations of ΔT have an amplitude which is much smaller than the final value of the response to $\Delta T_i = step(t)$

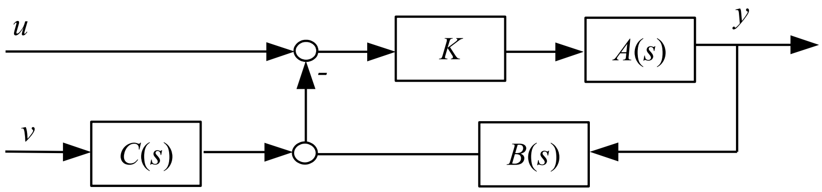
Question 4

Considering the following block diagram

$$A(s) = \frac{100}{s}$$

$$B(s) = \frac{1}{1+s}$$

$$C(s) = \frac{40}{1+10s+10s^2}$$



4.1 Compute the transfer functions between the inputs u and v and the output y.

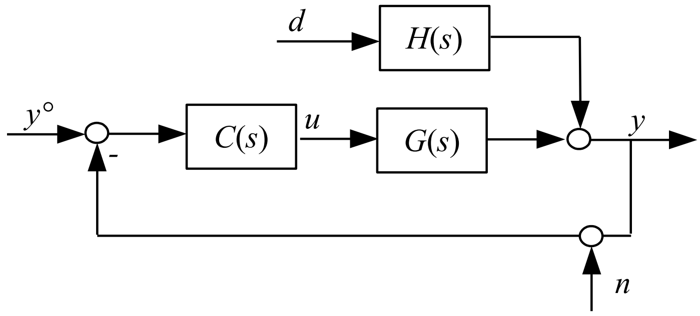
4.2 Determine for which values of the parameter K the system shows damped, exponentially decaying oscillations in response to step changes of the inputs.

Question 5

Consider the following control system, where the unit of time constants is the second:

$$G(s)=\frac{500(1-10s)}{s(1+20s)}$$

$$H(s)=\frac{150}{s}$$



5.1 Design a PI or PID controller with a bandwidth of 0.01 rad/s and at least 50° phase margin.

5.2 Plot the qualitative diagrams of the response of the controlled output y to a step change of the set point $y°$.

5.3 Design a feed-forward disturbance compensator for the system. Is there any upper bound to the bandwidth where the compensator can be effective?

5.4 How is the dynamic performance of the system affected if the gain of G(s) turns out to be 1000 instead of 500?

5.5 Assume $n = sin(10t)$. Is it possible to modify the controller designed at point 5.1 so that the set point tracking and the rejection of the disturbance d is unchanged, but the amplitude of the oscillations of the manipulated variable u is drastically reduced?