Control Systems

(Prof. Casella)

Midterm Exam – May 7th, 2015

Name:...

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Reg. Number:...............................

Signature:......................................

Notices:

- This booklet is comprised of 6 sheets – Check that it is complete and fill in the cover.
- Write your answers in the blank spaces with short arguments, including only the main steps in the derivation of the results.
- You are not allowed to leave the classroom unless you hand in the exam paper or withdraw from the exam.
- You are not allowed to consult books or lecture notes of any kind.
- Please hand in only this booklet at the end of the exam – no loose sheets.
- The clarity and order of your answers will influence how your exam is graded.

Question 1

Define the transfer function of a linear time-invariant system described by the following state-space equations and state its main properties in the case of single-input, single-output system.

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Question 2

Assess the stability of the systems described by the following transfer functions:

	Asym. stable	Simply stable	Unstable
$\frac{s-1}{s^2-10s+20}$	❑	❑	❑
$\frac{s+3}{s^2+1}$	❑	❑	❑
$\frac{s}{(s+1)^3}$	❑	❑	❑
$\frac{s}{(s^2+1)^3}$	❑	❑	❑
$\frac{5}{s^3+s^2}$	❑	❑	❑
$\frac{1}{s(s-3)}$	❑	❑	❑
$\frac{1}{s(s+10)}$	❑	❑	❑
$\frac{1}{s(s+1)^2}$	❑	❑	❑
$\frac{1}{(s-1)^2(s+1)^2}$	❑	❑	❑

Question 3

Consider a room of volume V with a forced ventilation system supplying a mass flow rate w_i of fresh air and extracting a mass flow rate w_o of room air of density ρ. The room occupants produce a mass flow rate w_c of CO_2 (neglecting perspiration and oxygen consumption for simplicity). Assuming the air is well-mixed, the total mass balance and CO_2 mass balance equations read:

$$\frac{d\rho V}{dt} = w_i + w_c - w_0$$

$$\frac{d\rho Vx}{dt} = w_i x_i + w_c - w_0 x$$

where x_i and x the mass fractions of CO_2 in fresh air and room air. By multiplying the first equation by x and subtracting it from the second, the following equations are finally obtained:

$$V\frac{d\rho}{dt} = w_i + w_c - w_0$$

$$\rho V\frac{dx}{dt} = w_i(x_i - x) + w_c(1 - x)$$

3.1 Write the system equations in standard state-space form, considering the fresh air, CO_2, and room air mass flows w_i, w_c, w_o as inputs and the CO_2 mass fraction x as output.

3.2 Compute the equilibrium conditions of the system. You may simplify the results by assuming that typical CO_2 mass fractions are several orders of magnitude smaller than one.

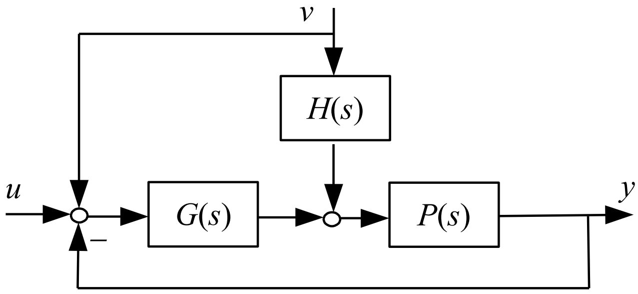
3.3 Write the linearized system equations in the neighbourhood of the found equilibria.

3.4 Compute the transfer functions between the inputs Δw_i and Δw_c and the output Δx, write them in gain/time constant form and plot their step response diagrams.
(Hints: only one state equation is involved; consider the equilibrium condition carefully when evaluating the transfer functions)

3.5 Discuss how the speed of response of these transfer functions change if the equilibrium value of the fresh air flow is doubled, and if the equilibrium value of the CO_2 dioxide flow is doubled.

Question 4

4.1 Compute the transfer functions between the inputs u and v and the output y of the following block diagram, writing them in gain/time constants form.



$$G(s) = 10\frac{1+2s}{2s}$$

$$P(s) = \frac{2}{(1+2s)(1+10s)}$$

$$H(s) = \frac{1}{1+s}$$

4.2 What happens to the stability of the found transfer functions if the sign of the feedback signal entering the summation node on the left is changed from minus to plus?

Question 5

Draw the Bode plots of the frequency response of the system $G(s)=10\frac{1-s}{(1+s)(1+0.01s+0.01s^2)}$

Question 6

Design a filter $F(s)$ such that:

- the amplitudes of the harmonic components in the input with frequency $\omega \ll \omega_0$ or $\omega \gg \omega_0$ are left unchanged in the asymptotic response of the output
- the harmonic component at frequency $\omega = \omega_0$ is completely removed from the asymptotic response of the system output

where ω_0 is a known design parameter *(Hint: use complex poles and zeros).*