Control Systems

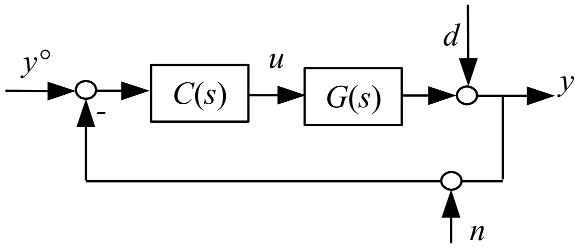
(Prof. Casella)

Final Exam – June 26th, 2015

Answers sheets

Question 1

Consider the control system shown in the figure. Define the *control sensitivity* transfer function and explain why is it important to assess the system performance. Finally, explain how to approximate its frequency response assuming that the loop transfer function satisfies the pre-conditions of Bode's criterion.



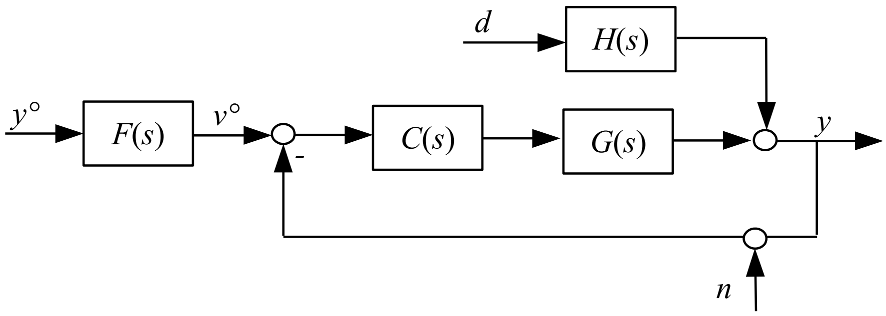
The sensitivity function is defined as $Q(s) = C(s)/(1+C(s)G(s))$.

Disregarding the sign, it is the transfer function between each of the inputs $y°$, d, and n, and the manipulated variable u, so it allows to understand how sensitive the latter is to all the disturbances acting on the system.

It is possible to approximate $Q(j\omega) \approx 1/G(j\omega)$ for $\omega \ll \omega_c$, and $Q(j\omega) \approx C(j\omega)$ for $\omega \gg \omega_c$

Question 2

Draw the block diagram of a 2-degrees-of-freedom controller using set-point pre-filtering. Briefly explain the design criteria for the controllers.



The feed-back controller $C(s)$ must be designed to meet the specifications regarding disturbance rejection (both from d and from n). The pre-filter $F(s)$ must then be designed so that

$$\frac{Y(s)}{Y°(s)} = F(s)\frac{C(s)G(s)}{1+C(s)G(s)}$$

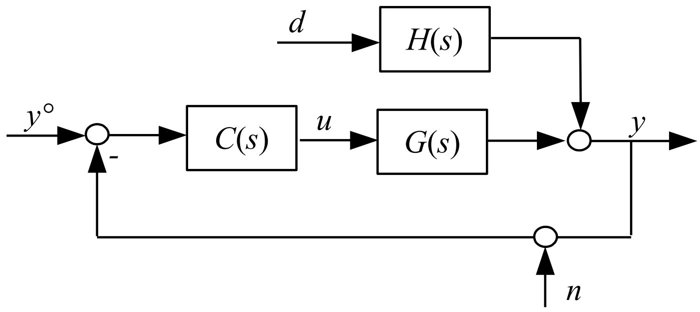
has the required dynamic behaviour.

Question 3

Consider the following control system (time constants are given in seconds), where $C(s)$ is a real PID controller with $K_p = 50$, $T_i = 100$, $T_d = 20$, $N = 3$:

$$G(s)=\frac{0.15}{(1+100s)(1+20s)(1+5s)(1+2s)}$$

$$H(s)=\frac{4}{s(1+100s)}$$



3.1 Compute the steady-state errors corresponding to unit step changes of the set point $y°$ and of the disturbance d.

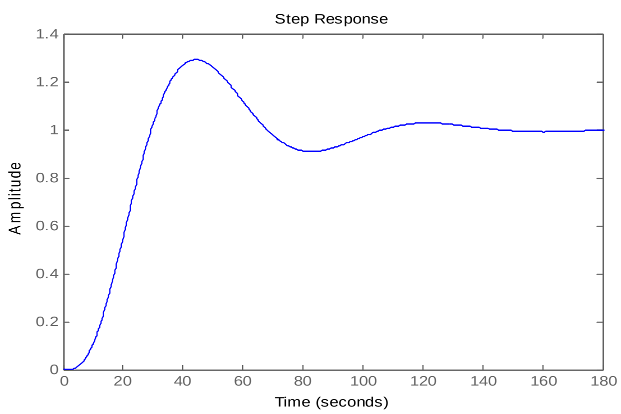
$$e_{\infty y°}=\lim_{s\to 0}\frac{C(s)G(s)}{1+C(s)G(s)}=0$$

$$e_{\infty d}=lim-¿s\rightarrow 0\frac{H(s)}{1+C(s)G(s)}=-53.3$$

3.2 Compute the crossover frequency and the phase margin of the control system. Then, plot an approximated diagram of the step response of the controlled variable y to a step change of the set point $y°$.

Considering the asymptotic magnitude plot, $\omega_c = 0.075$; as there are two poles at slightly higher frequency, this value is slightly overestimated. The corresponding phase margin is $\varphi_m = 35°$, which is slightly underestimated, as the actual crossover frequency will be somewhat lower, corresponding to a somewhat higher phase margin.

The exact values are in fact $\omega_c = 0.065$ and $\varphi_m = 41°$.



3.3 Compute the asymptotic amplitudes of the oscillations of the manipulated variable u and of the controlled variable y corresponding to a sinusoidal feed-back disturbance n having unit amplitude and a period of 0.1 seconds.

The input signal has an angular frequency $\omega = 2\pi/T = 62.8$ rad/s >> ω_c.

Regarding the manipulated variable u, the amplification factor is
$|Q(j\omega)| \approx |C(j\omega)| \approx K_p N = 150$,
which will also be the asympotic amplitude of the controlled variable oscillations.

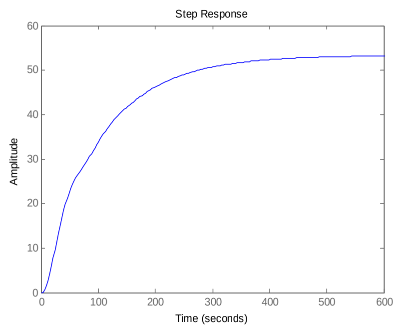
Regarding the controlled variable y, the amplification factor is
$|T(j\omega)| \approx |C(j\omega)G(j\omega)| \approx K_p N \cdot 0.15 / (100\omega \cdot 20\omega \cdot 5\omega \cdot 2\omega) = 7.2 \cdot 10^{-11}$
which will also be the asympotic amplitude of the controlled variable oscillations.

3.4 Plot an approximated diagram of the response of the controlled variable y to a step change of the disturbance d.

The transfer function between d and y is

$$H(s)S(s) \approx \frac{4}{s(1+100s)} \frac{13.3s}{1+\frac{2 \cdot 0.35}{0.075}+\frac{1}{0.075^2}s^2} \approx \frac{53.3}{(1+100s)(1+\frac{2 \cdot 0.35}{0.075}+\frac{1}{0.075^2}s^2)}$$

The dominant, slowest dynamics is a first-order dynamics with a settling time of about 500 s.



3.5 Explain how the static and dynamic system performance changes if the actual gain of $G(s)$ turns out to be ½ of its design value, using the same controller.

Regarding the static performance, as the loop gain is halved, the steady-state error is doubled.

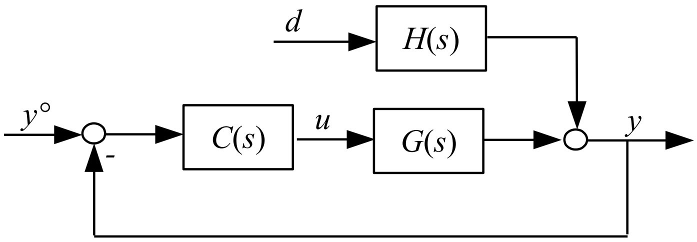
The crossover frequency is halved to 0.0375, while the phase margin increases to 62°. As a consequence, the step response of the controlled output to a step change of the set point no longer oscillates and has a settling time of about 135 s, while the settling time of the response to a step disturbance is still about 500 s, as the dominant pole remains the real pole with time constant T_i.

Question 4

Consider the following control system, where the unit of time constants is the second:

$$G(s)=0.1\frac{e^{-10s}}{(1+3s)(1+0.5s)}$$

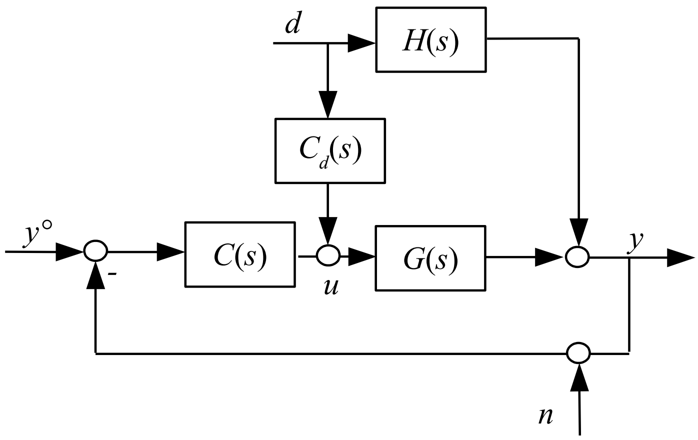
$$H(s)=0.5\frac{e^{-10s}}{1+3s}$$



4.1 Design a PI or PID controller with a bandwidth of 0.04 rad/s and a phase margin of at least 60°.

A PI controller is sufficient in this case. One possible tuning is Kp = 1.33, Ti = 3, which gives the required bandwidth and a phase margin of 66°.

4.2 Design a disturbance compensator to improve the disturbance rejection in the frequency range 0–0.4 rad/s



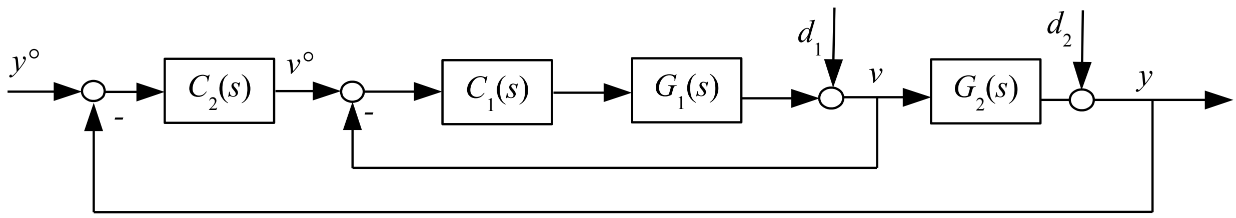
$$C_d(s)=-H(s)/G(s)=5(1+0.5s)$$
$$C_{d1}(s)=-5$$
$$C_{d2}(s)=-\frac{5}{1+2.5s}$$

The ideal compensator $C_d(s)$ is improper. A simple static compensator $C_{d1}(s)$ provides effective compensation up to about 2 rad/s. If that is not required and the measurement of d is noisy, it is possible to limit the bandwidth to the design requirement by using $C_{d2}(s)$ instead.

Notice that there are no limitations to the bandwidth due to the delay in the plant, since both transfer functions have the same delay. This makes feed-forward disturbance compensation very effective in this case.

Question 5

5.1 Draw the block diagram of a cascaded controller and briefly discuss its design criteria



The inner controller $C_1(s)$ is designed considering the plant transfer function $G_1(s)$ only, with a large bandwidth. The outer controller $C_1(s)$ uses the set-point $v°$ as a virtual manipulated variable, and is tuned with a much smaller bandwidth – this makes it possible to approximate the transfer function between $v°$ and v as 1, so that the outer controller is designed considering the plant transfer function $G_2(s)$ only.

5.2 Discuss what are the advantages and disadvantages of this control strategy compared to a standard feedback controller applied to the same plant.

The main advantages are a much better rejection of disturbance d_1, and a higher robustness w.r.t. uncertainty in $G_1(s)$ when designing the "difficult" outer control loop. The main disadvantage is the need for an extra sensor to measure the intermediate output variable v.